Chapter 6

Magnetoconductance of the free 2DEG

The experimental realization of a free two dimensional electron gas (2DEG) is outlined. The Shubnikov-de-Haas oscillations (SdH) in its longitudinal resistivity are reproduced by the semiclassical Kubo formula, but the plateaus in the Hall resistivity, i. e. the integer quantum Hall effect (QHE), are not. The description of the QHE succeeds by including a specific higher-order $\hbar$ term originating from the level density. The corresponding correction is derived for general systems.

Contents

The free two dimensional electron gas (2DEG) is predisposed as a test system for the semiclassical Kubo formula Eq. (5.4). The longitudinal conductivity in the presence of a transverse magnetic field B was already evaluated by the authors of the semiclassical Kubo formula [63, 41]. In the following sections, both the longitudinal and the Hall conductivity for the free 2DEG are derived. The resulting description is also valid for particles with spin. For those, n_s denotes the electron density per spin orientation, i. e. $n_s = 2(S+1/2)n_e$. For spin-less particles, n_s is given by the total electron sheet density n_e.

6.1 Two dimensional electron gas

The electronic bands of semiconductors bend at interfaces. In a suitable designed heterostructure (e. g. GaAs/GaAlAs), this leads to a narrow, triangular region at the interface where the conduction band is below the Fermi energy. E_F can be chosen so that only the lowest eigenstate of this well is occupied. For sufficiently low thermal energies higher states are energetically unaccessible, so that the corresponding degree of freedom is blocked. From a quantum mechanical point of view, such a system is truly two-dimensional. Fig. 6.1 illustrates this situation.

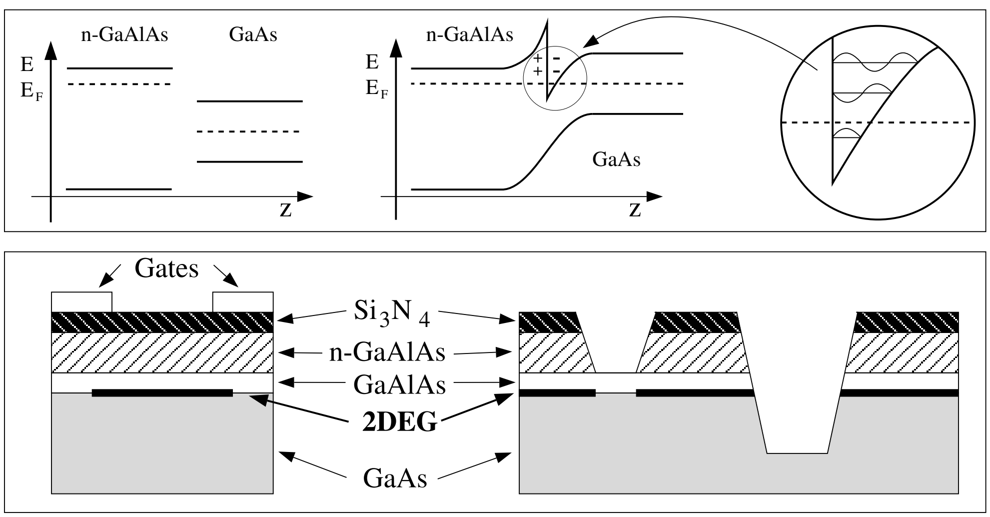

Figure 6.1: *Upper part: The mechanism leading to a 2DEG at the interface. Lower part: An undoped spacer layer between GaAs and n-GaAlAs reduces impurity scattering at the donors. The 2DEG can additionally be laterally confined by electrostatic gates (left), shallow etching (middle) or deep etching (right).*

Implementing the donors at a distance from the interface extremely reduces the impurity scattering. At low temperatures, where electron-phonon scattering can be neglected, this is nevertheless the dominant scattering mechanism. This comes about as semiconductors can nowadays be produced with extremely low contaminations and lattice defects. The latter is facilitated by the nearly identical lattice constants of GaAs and GaAlAs. For these reasons, the mobility of those devices can be extremely high. The mean-free path in state-of-the-art samples exceeds 10μm.

An additional lateral confinement of the 2DEG is possible either by etching or by applying electrostatic gates. By electron beam lithography structures in the 10nm regime can be defined. This is comparable to the Fermi wavelength, which is typically of the order of some 10nm.

6.2 The classical conductivity

In the classical picture, an external electric field accelerates the electrons. Due to impurity collisions, they acquire an average drift velocity $\vec{v}_d = -\mu \vec{E}$. The mobility μ is related to the scattering time τ_s via $\mu = e\tau_s/m^\star$, and the mean-free path is given by $\ell = v_F \tau_s$. The classical magnetoconductivity can be derived using the Einstein relation

$$\underline{\widetilde{\sigma}} = e^2\, g(E_F)\, \underline{D}\,, \tag{6.1}$$

where g denotes the level density and $\underline{D}$ the diffusion tensor. $\underline{D}$ can be evaluated within linear response, leading to [103]

$$D_{ij} = \int_0^\infty dt\, \langle v_i(t)\; v_j(0)\rangle\;, \tag{6.2}$$

where the brackets denote an average over the Fermi surface. This finally leads to the Drude conductivity tensor (for a detailed derivation see e. g. Ref. [99])

$$\underline{\widetilde{\sigma}} = \frac{\sigma_0}{1+(\omega_c\tau_s)^2}\begin{pmatrix} 1 & -\omega_c\tau_s \\ \omega_c\tau_s & 1 \end{pmatrix} \quad ; \quad \sigma_0 = \frac{n_s e^2 \tau_s}{m^\star} = n_s e \mu\;. \tag{6.3}$$

The symmetry of the system enforces $\widetilde{\sigma}_{xx} = \widetilde{\sigma}_{yy}$ and $\widetilde{\sigma}_{xy} = -\widetilde{\sigma}_{yx}$. Using Eq. (5.10), the resistivity tensor $\underline{\widetilde{\rho}}$ is given by

$$\underline{\widetilde{\rho}} = \rho_0 \begin{pmatrix} 1 & \omega_c\tau_s \\ -\omega_c\tau_s & 1 \end{pmatrix} \quad ; \quad \rho_0 = \frac{1}{\sigma_0} = \frac{m^\star}{n_s e^2 \tau_s}\;. \tag{6.4}$$

The classical longitudinal resistivity $\widetilde{\rho}_{xx} = \rho_0 = m^\star/(n_e e^2 \tau_s)$ is independent of the magnetic field. Experimentally, the classical limit is recovered in the low-field regime. Therefore the measurement of $\widetilde{\rho}_{xx}|_{B=0}$ is a convenient way to determine the mobility μ (and thus the scattering time τ_s). The Hall resistivity $\widetilde{\rho}_{xy} = B/(en_s)$ is proportional to the magnetic field, which is consistent with the usual definition of the Hall resistance.

In analogy to the smooth part of the level density $\widetilde{g}$ considered above, the classical (smooth) part of the conductivity (resistivity) of the free 2DEG according to Eq. (6.3) (Eq. (6.4)) is denoted with a tilde.

6.3 Leading order in $\hbar$

The trace formula for the oscillating part of the conductivity given in Sec. 5.5 is, just as the Gutzwiller trace formula, only valid for isolated periodic orbits. An extension to the case of the free 2DEG with its two-dimensional translational symmetry is possible using an approach analog to Creagh and Littlejohn's treatment of the level density for systems with continuous symmetries (see Sec. 2.3, especially Eq. (2.15)). Alternatively one can proceed as for the calculation of the level density associated with the cyclotron orbits (cf. Sec. 4.2.1.3). Both approaches reduce the problem to a one dimensional harmonic oscillator with an additional factor

$$\kappa = V \cdot \frac{eB}{2\pi\hbar} \tag{6.5}$$

from integration over the symmetry. The cyclotron orbit is the only primitive periodic orbit of the system, so that the sum over all orbits in the trace formula reduces to the sum over its repetitions. The velocity-velocity correlation of the primitive periodic orbit according to Eq. (5.7) can be calculated analytically, resulting in

$$\mathcal{C}_{xx} = R_c^2 \pi \frac{\omega_c \tau_s}{1 + (\omega_c \tau_s)^2} \quad \text{and} \quad \mathcal{C}_{xy} = R_c^2 \pi \frac{(\omega_c \tau_s)^2}{1 + (\omega_c \tau_s)^2} , \tag{6.6}$$

with the cyclotron frequency $w_c = eB/m^\star$ and the cyclotron radius $R_c = \hbar/eB \cdot \sqrt{4\pi n_s}$. Inserting all this into Eq. (5.4) and denoting the period of the primitive orbit with $T_0 = 2\pi/\omega_c$ results in

$$\begin{aligned}
\delta\sigma_{xx} &= 2 \frac{\sigma_0}{1 + (\omega_c \tau_s)^2} \sum_{p=1}^{\infty} R(pT_0) e^{-p^2 T_0 / 2\tau_s} \cos\left[2\pi p \left(\widetilde{E} - \frac{1}{2}\right)\right] \\
\delta\sigma_{xy} &= -2 \frac{1}{\omega_c \tau_s} \frac{\sigma_0}{1 + (\omega_c \tau_s)^2} \sum_{p=1}^{\infty} R(pT_0) e^{-p^2 T_0 / 2\tau_s} \cos\left[2\pi p \left(\widetilde{E} - \frac{1}{2}\right)\right] .
\end{aligned} \tag{6.7}$$

The relation between σ_{xx} and σ_{xy} is remarkable: Both the classical contributions according to Eq. (6.3) and the oscillating parts of Eq. (6.7) are proportional to each other, but with inverse factors:

$$\widetilde{\sigma}_{xy} = -\omega_c \tau_s \cdot \widetilde{\sigma}_{xx} , \quad \text{and} \quad \delta\sigma_{xy} = -1/(\omega_c \tau_s) \cdot \delta\sigma_{xx} . \tag{6.8}$$

In the strong field limit the classical Hall conductivity thus dominates over the longitudinal conductivity ($\widetilde{\sigma}_{xy} \gg \widetilde{\sigma}_{xx}$), and the quantum oscillations in σ_{xy} are suppressed compared to the oscillations in σ_{xx} ($\delta\sigma_{xy} \ll \delta\sigma_{xx}$).

On Landau levels, the normalized energy $\widetilde{E}$ in Eq. (6.7) is identical to the Landau quantum number. For spin-less particles it is therefore given by $\widetilde{E} = E_F/(\hbar\omega_c) = 2\pi\hbar n_s/(eB)$. Including spin leads to

$$\widetilde{E} = \frac{E_F}{\hbar\omega_c} + s g^\star \frac{1}{2} \frac{m^\star}{m_e} , \tag{6.9}$$

with the spin quantum number s and the Landé g-factor $g^\star$ of the material. For the 2DEG in GaAs, $s = \pm 1/2$ and $g^\star m^\star / m_e \approx -0.0293$. This corresponds to a spin splitting of $\approx 1.5\%$ of the Landau level separation, which usually cannot be detected. For InAs, in contrast, $g^\star m^\star / m_e \approx 0.338$, leading to a separation of the two spin peaks of $\approx 17\%$ of the Landau level distance. This explains why in GaAs/GaAlAs heterostructures the spin is generally neglected. Including spin, the contributions of the spin-subsystems to $\delta\underline{\sigma}$ have to be added. As obvious from Eq. (6.9), the inclusion of spin only leads to a shift of the Landau levels. Since within this approach no additional spin-related effects are included, the discussion of Eq. (6.7) can be restricted to spin-less particles without loss of generality.

The total resistivity $\underline{\rho} = \underline{\sigma}^{-1}$ is found by adding the classical part according to Eq. (6.4) and the semiclassical approximation of the quantum oscillations of Eq. (6.7), i. e.

$$\underline{\sigma} = \underline{\widetilde{\sigma}} + \delta\underline{\sigma} . \tag{6.10}$$

For the longitudinal conductivity σ_{xx}, the quantum mechanical self-consistent Born approximation for short-range scatters [9, 87] in the low-field regime $\omega_c \tau_s < 1$ is equivalent to this semiclassical result.

In Fig. 6.2 the result of Eq. (6.7) for ρ_{xx} and ρ_{xy} is shown for a system with electron density $n_s = 1.0 \cdot 10^{16}\mathrm{m}^{-2}$ and mobility $\mu = 100\mathrm{m}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$ at a temperature of 10K. The classical resistivity (solid) is compared to the semiclassical description (dashed). The quantum oscillations to ρ_{xx} are seen to be an important correction in large fields. They give rise to the Shubnikov-de-Haas (SdH) oscillations.

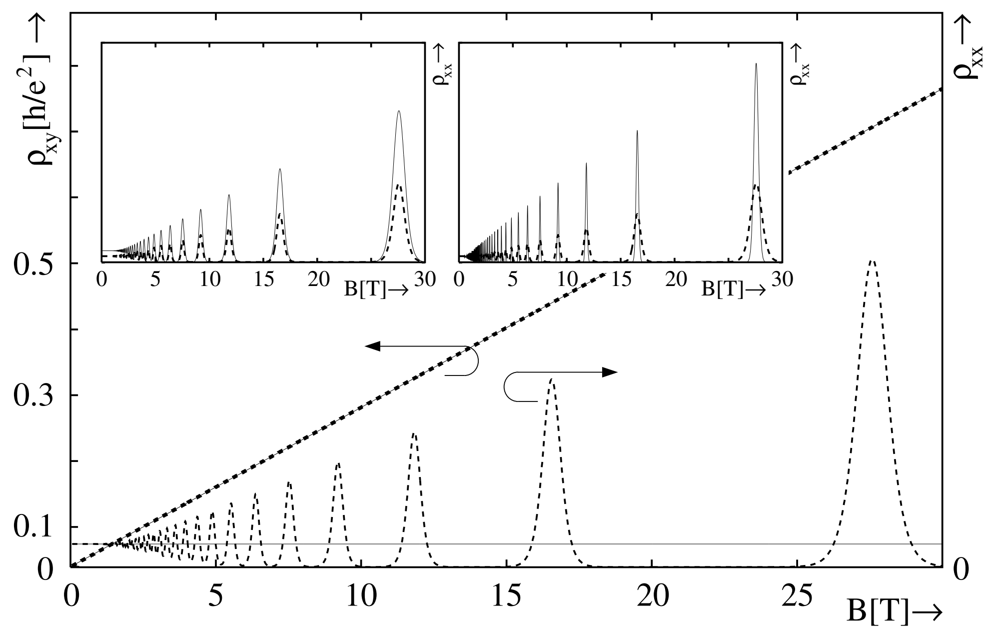

Figure 6.2: *The diagonal and the Hall resistivity of the free 2DEG. Solid: classical result of Eq. (6.4); dashed: semiclassical results according to Eqs. (6.10, 6.7, 6.4). The SdH oscillations in the longitudinal resistivity are well reproduced, but the QHE is not recovered in the semiclassical approximation. Insets show the influence of mobility (left) and temperature (right) on ρ_{xx}. Dashed: results of the central graphic ($\mu = 100\mathrm{m}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$, $T = 10\mathrm{K}$); solid: $\mu = 50\mathrm{m}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$ (left inset), $T = 3\mathrm{K}$ (right inset).*

The dependence on the scatterer density is illustrated in the left inset. The dashed line repeats the result of the main graphic with $\mu = 100\mathrm{m}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$, whereas the solid line corresponds to $\mu = 50\mathrm{m}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$. The zero field resistance is, as expected from Eq. (6.4), inverse proportional to the mobility. The amplitude of the SdH oscillations increases with lower mobility, and their relative width remains essentially unchanged. This is not consistent with the general accepted picture of the SdH oscillations. An increased scatterer density extends the region of localized states between the Landau levels, pushing the mobility edges closer to the Landau levels. This leads to sharper peaks in ρ_{xx}. The localization of states is semiclassically due to periodic orbits which include scattering events. As pointed out in Sec. 5.2, those have been neglected in the derivation of the semiclassical Kubo formula. Therefore the semiclassical approximation for the diffusive limit can not be expected to describe the SdH line widths correctly.

The right inset shows the influence of temperature. The dashed line repeats the result of the central graphic for $T = 10\mathrm{K}$, whereas the solid line corresponds to $T = 3K$. The SdH oscillations get sharper for lower temperatures. High temperatures lead, as expected, to the classical limit $\rho_{xx} = \rho_0$. This is the correct temperature dependence.

In conclusion, the longitudinal resistivity of the 2DEG is well approximated by the semiclassical Kubo formula. It only fails to reproduce the correct dependence of the peak widths on the mobility. This is due to the neglect of periodic orbits including scattering.

In contrast to the successful description of ρ_{xx}, the semiclassical approximation of the Hall resistivity is inadequate. The semiclassical correction to the off-diagonal resistivity Eq. (6.7) plotted in Fig. 6.2 is completely negligible. It does not reproduce the integer quantum Hall effect (QHE).

6.4 $\hbar$ correction from the level density

The failure of the semiclassical description for ρ_{xy} might be due to the restriction to leading order in $\hbar$. Higher-order corrections can be implemented by going back to the quantum mechanical formula for the linear transport properties proposed by Středa [75]:

$$\begin{aligned} V \cdot \sigma_{xx} &= \pi e^2 \hbar \,\mathrm{Tr}[v_x \delta(E-H) v_x \delta(E-H)] \\ V \cdot \sigma_{xy} &= e\,\frac{\partial N(E,B)}{\partial B} + \frac{i}{2} e^2 \hbar \,\mathrm{Tr}[v_x G^+(E) v_y \delta(E-H) - v_x \delta(E-H) v_y G^-(E)] \,. \end{aligned} \tag{6.11}$$

Here $G^{\pm}$ denotes the advanced and retarded Green's function, respectively. The second term of σ_{xy} is analog to the expression for σ_{xx}. This well approximated by the semiclassical Kubo formula, indicating that higher-order corrections to this term are irrelevant.

In the following, a higher-order correction to $\partial N/\partial B$ shall be derived. It should not be restricted to the free 2DEG, so that the general form of a semiclassical level density is chosen as starting point:

$$\delta g(E) = \frac{1}{\hbar^{(k+2)/2}} \sum_{\mathrm{po}} A_{\mathrm{po}}\, T_{\mathrm{ppo}} \cos\left(\frac{S_{\mathrm{po}}}{\hbar} - \mu_{\mathrm{po}}\frac{\pi}{2}\right) , \tag{6.12}$$

The damping terms due to temperature and impurities as well as other prefactors are included in the amplitudes A_{po} for notational convenience. The volume term of the Thomas-Fermi level density, which gives the leading contribution to the smooth part of $g(E)$, is independent of the magnetic field. Therefore only the oscillating part of the level density contributes to $\partial N/\partial B$. Using

$$\delta N(E,B) = \int_0^{E_F} \delta g(E,B)\, dE \,, \tag{6.13}$$

the semiclassical approximation for the first term in Eq. (6.11) can be expressed as

$$\begin{aligned} \delta\sigma_{xy}^{I} &:= e\,\frac{\partial N(E,B)}{\partial B} = e\,\frac{\partial[\delta N(E,B)]}{\partial B} \\ &\approx -\frac{e}{\hbar^{(k+2)/2}} \sum_{\mathrm{po}} \frac{1}{p}\frac{\partial}{\partial B} \int_0^{E_F} A_{\mathrm{po}}(E,B) \frac{\partial S_{\mathrm{po}}}{\partial E} \cos\left(\frac{S_{\mathrm{po}}(E,B)}{\hbar} - \mu_{\mathrm{po}}\frac{\pi}{2}\right) dE \\ &= -\frac{e}{\hbar^{(k+2)/2}} \sum_{\mathrm{po}} \frac{1}{p}\frac{\partial}{\partial B} \int_{S(0,B)}^{S(E_F,B)} A_{\mathrm{po}}(S,B) \cos\left(\frac{S_{\mathrm{po}}(S,B)}{\hbar} - \mu_{\mathrm{po}}\frac{\pi}{2}\right) dS \,. \end{aligned} \tag{6.14}$$

In the second line it was used that the period of an orbit is given by $T_{\rm po} = p \cdot T_{\rm ppo} = p \cdot \partial S_{\rm ppo}/\partial E$, where p denotes the winding number. The last step assumed that the action $S(E,B)$ is invertible[1], so that $E(S,B)$ is uniquely defined for all S and B. The integral over S can be performed by subsequent partial integration. The partial derivative with respect to B acts both on the semiclassical amplitude of the orbit and on the action.[2] Taking these derivatives and sorting the terms in powers of $\hbar$ finally leads to

$$\delta\sigma^I_{xy} \approx \frac{-e}{\hbar^{(k+2)/2}} \sum_{\rm po} \frac{1}{p} \left[A_{\rm po} \frac{\partial S_{\rm po}}{\partial B} \cos\left(\frac{S_{\rm po}}{\hbar} - \mu_{\rm po}\frac{\pi}{2} \right) \right]_{E=0}^{E=E_F} + \delta\sigma^\hbar_{xy} \,, \tag{6.15}$$

$$\begin{aligned} \delta\sigma^\hbar_{xy} = \frac{-e}{\hbar^{(k+2)/2}} \sum_{\rm po} \frac{1}{p} \Bigg[\sum_{i=1}^{\infty} \hbar^i \cos\left(\frac{S_{\rm po}}{\hbar} - \mu_{\rm po}\frac{\pi}{2} + i\frac{\pi}{2} \right) \times \\ \times \left\{ \left(\frac{d^i}{dS^i} A_{\rm po}(S,B) \right) \frac{\partial S_{\rm po}}{\partial B} - \frac{\partial}{\partial B}\left(\frac{d^{i-1}}{dS^{i-1}} A_{\rm po}(S,B) \right) \right\} \Bigg]_{E=0}^{E=E_F} . \end{aligned}$$

The leading order in $\hbar$ is given by the first term of $\delta\sigma^I_{xy}$ in Eq. (6.15). If the contribution of the lower integration limit vanishes, this reproduces the term $1/e\, \partial S/\partial B$ of the semiclassical Kubo formula Eq. (5.4). $\delta\sigma^\hbar_{xy}$ gives a series of $\hbar$-corrections.

The starting point of this derivation is given by the semiclassical level density, which by itself is only valid in leading order in $\hbar$. Higher-order corrections to δg entail additional terms to Eq. (6.15). If the semiclassical approximation of the level density of the system is good, $\delta\sigma^\hbar_{xy}$ as given in Eq. (6.15) contains the dominant corrections.

For the free 2DEG this condition is fulfilled, since the semiclassical approximation of its level density is exact (cf. Sec. 4.2.1.3). For this system all $\hbar$ corrections to $\delta\sigma^I_{xy}$ are included in Eq. (6.15).[3] The relevance of the $\hbar$ corrections in Eq. (6.15) for the conductivity tensor of the free 2DEG shall now be discussed.

By writing the prefactors of the level density Eq. (4.11) as a product of dS/dE and an amplitude, Eq. (6.15) can be applied to the free 2DEG.[4] Including explicitly finite temperature and impurities by appropriate damping terms, the result reads

$$\begin{aligned} \delta\sigma^I_{xy} &= \frac{2en_s}{B} \sum_{p=1}^{\infty} R(pT_0) e^{-p^2 T_0/2\tau_s} \cos\left[2\pi p \left(\widetilde{E} - \frac{1}{2} \right) \right] + \delta\sigma^\hbar_{xy} \,, \\ \delta\sigma^\hbar_{xy} &= -\frac{e^2}{h\pi} \sum_{p=1}^{\infty} \frac{1}{p} R(pT_0) e^{-p^2 T_0/2\tau_s} \sin\left[2\pi p \left(\widetilde{E} - \frac{1}{2} \right) \right] . \end{aligned} \tag{6.16}$$

The period of the primitive cyclotron orbit is given by $T_0 = 2\pi m/(eB)$. The first term of $\delta\sigma^I_{xy}$ is the leading-order contribution in $\hbar$[5]. It is already included in the trace formula Eq. (6.7).[6] Only the first term of Eq. (6.15) is nonzero, so that for the 2DEG only

[1] The general case is that $S(E,B)$ can only be piecewise inverted. The following derivation can be extended to this situation. This introduces only the inconvenience to notate the correct branch or, if necessary, the sum over the relevant branches.

[2] If the Maslov index $\mu_{\rm po}$ is replaced by the reflection phase φ_R, a third term shows up in the following calculation, since φ_R depends on B and E.

[3] The other terms of Eq. (6.11), of course, may give rise to additional higher-order contributions.

[4] Alternatively, $\delta\sigma^I_{xy}$ can be directly evaluated from δg of Eq. (4.11).

[5] Note that the sheet density contains powers of $\hbar$, since $n_s = E_F m/(2\pi\hbar^2)$.

[6] Note that $\frac{en_s}{B} = \frac{\sigma_0}{w_c\tau_s}$ and $\frac{1}{w_c\tau_s} - \frac{w_c\tau_s}{1+(w_c\tau_s)^2} = \frac{1}{w_c\tau_s[1+(w_c\tau_s)^2]}$ to compare with Eq. (6.7).

corrections in second leading order $\hbar$ show up. The prefactor of $\delta\sigma_{xy}^{\hbar}$ compared to the prefactor of $\delta\sigma_{xy}^{I}$ increases linearly in B. Therefore this correction, although of lower order in $\hbar$, becomes dominant in strong fields.

The influence of this $\hbar$ correction is presented in Fig. 6.3. The solid line indicates the semiclassical result in leading order in $\hbar$, as given above. The dashed line includes the correction of $\delta\sigma_{xy}^{\hbar}$. The semiclassical trace formula now reproduces the plateaus in the Hall resistance, i. e. the QHE. This shows that the quantum Hall effect is dominantly an effect of *second* leading order in $\hbar$. Its origin is the dependence of the period $T_{\text{ppo}} = dS/dE$ on the magnetic field.

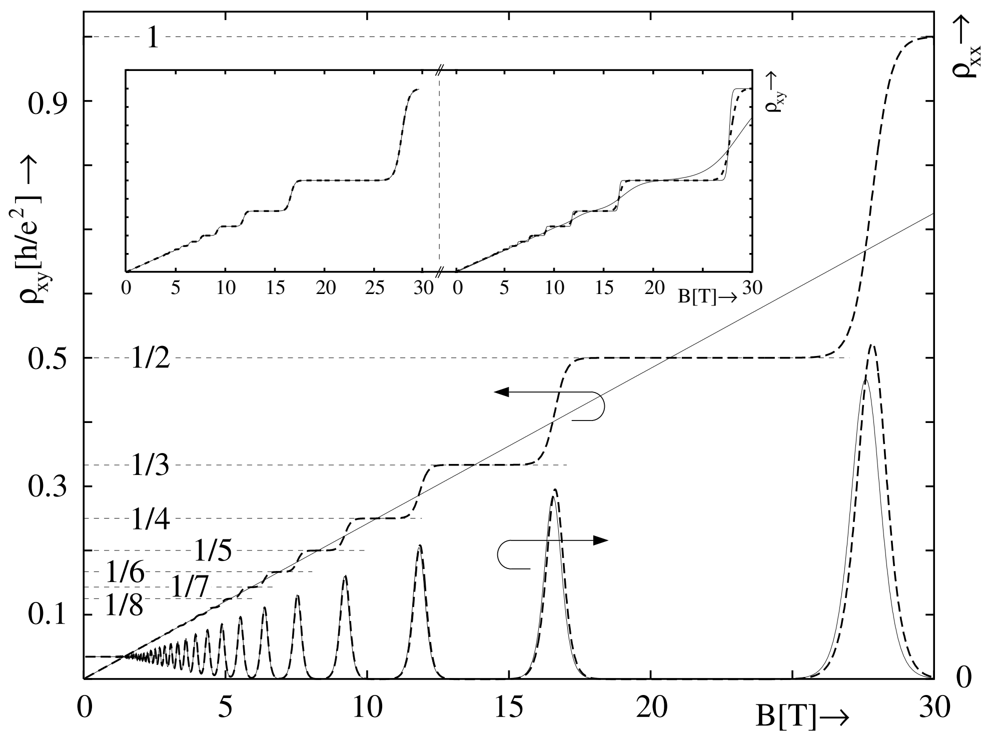

Figure 6.3: *The longitudinal and the Hall resistivity for the free 2DEG. Solid: semiclassical result in leading order in $\hbar$ as in Fig. 6.2; dashed: semiclassical result including the $\hbar$ correction of Eq. (6.16). The correction to the longitudinal resistance is negligible. The quantized conduction, however is purely an effect of second leading order in $\hbar$. Insets show the influence of mobility and temperature on ρ_{xy}. Dashed: results of the central graphic ($\mu = 100\text{m}^2\text{V}^{-1}\text{s}^{-1}$, $T = 10K$); solid: $\mu = 50\text{m}^2\text{V}^{-1}\text{s}^{-1}$ (left inset), $T = 3K$ and $T = 50K$ (right inset).*

The left inset shows the influence of the mobility on the QHE. The dashed line is a copy of the result shown in the the main graphic. It corresponds to $\mu = 100\text{m}^2\text{V}^{-1}\text{s}^{-1}$. The solid line shows the data for $\mu = 50\text{m}^2\text{V}^{-1}\text{s}^{-1}$. The Hall resistivity is hardly influenced by the amount of disorder. This does not agree with the established picture of the QHE, where the width of the localized states, and thus the width of the Hall plateaus, depends on the impurity concentration. The reason for this deficiency of the semiclassical approximation in the non-ballistic regime has already been given discussing the peak widths of the SdH oscillations above. Additionally, the validity of the present inclusion of finite free path-lengths is, as stated in Sect. 3.1, limited to the leading order in $\hbar$. It thus cannot be expected that this simple formalism reproduces the behaviour of higher-order $\hbar$ terms correctly.[7]

In the right inset of Fig. 6.3 the temperature dependence is depicted. The solid lines are calculated for $T = 3$K and $T = 50$K, respectively. For low temperatures the step-function is approached, whereas large temperatures smear the steps until the classical linear result is recovered. The temperature is therefore correctly included in the semiclassical approximation.[8]

In conclusion, the semiclassical Kubo formula successfully explains the longitudinal conductivity of the 2DEG, but fails for the Hall component. The $\hbar$ corrections according to Eq. (6.15) are the key ingredients for a semiclassical description of the Hall conductivity. The term in second leading order in $\hbar$ is responsible for the integer quantum hall effect. Including this term, the semiclassical Kubo formula explains both the Shubnikov-de-Haas oscillations and the QHE. It also reproduces the temperature dependence of these effects correctly. The approach is, however, limited to the ballistic regime. It therefore fails for effects that are related to localization, like the dependence of the QHE plateau width on the mobility. To describe these dependencies, periodic orbits which include scattering events would have to be taken into account.

The discussion of the Hall resistivity was restricted to the free 2DEG. The higher-order $\hbar$ corrections, however, were derived for arbitrary systems. They are not only relevant for samples which exhibit cyclotron-like orbits. For arbitrary systems, $\delta\sigma_{xy}^{\hbar}$ contains the relevant corrections if the semiclassical description of δg is sufficiently good, i. e. the higher-order $\hbar$ contributions to the *level density* can be neglected. Since this condition is frequently fulfilled, it is justified to include at least the second leading order term of Eq. (6.15) in all semiclassical descriptions of the Hall conductivity.

[7] Note that in this situation the procedure of Sect. 3.3 cannot be applied, since it starts out from the assumption that the line shape is known. The calculation of line shapes (or Hall plateaux widths) is thus obviously beyond the scope of this approach.

[8] This had to be expected, since the inclusion of finite temperature is exact for grand canonical systems as long as phonon scattering can be neglected (cf. Sect. 5.4).